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Risk-Return nexus in a GARCH-M Framework: Empirical Evidence from the South African Stock Market

By

Hlompho Morahanye

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Supervisor: DR Kwame Osei-Assibey

DECLARATION

I certify that the *minor dissertation/dissertation/thesis* submitted by me for the degree *Master's of Commerce (Financial Economics)* at the University of Johannesburg is my independent work and has not been submitted by me for a degree at another university.

Hlompho Morahanye



ABSTRACT

This paper studies the association between risk and returns in the Johannesburg Stock Exchange. In particular, the study is interested in modelling this relationship during periods of high volatility with special reference to the 2007-2009 financial crises. The objective is to highlight the effect that a high volatility period might have on the relationship. To achieve this objective, daily data for the market index, JSE Top 40 and the two JSE sectoral indices for the period 1/1/2004 to 3/5/2017 are used. The GARCH-M, E-GARCH-M and TARCH-M models and the same aforementioned models with dummy variables to account for two volatility regimes are used.

The CAPM prediction that the expected return on a stock above the risk-free rate is positive is not supported by the study. The tests conducted to examine the relationship observed that the risk premiums were either positive but insignificant, or negative and significant, which is inconsistent with the theory. The observed outcomes indicate that the risk premium is not necessarily positive, even after accounting for different regimes. These results are generally in line with observations made by other authors who investigated the relationship within the South African context. The findings of this paper are useful in financial decision-making, such as in providing investors with information on which sectors to invest in based on their risk appetite, as well as providing information regarding the performance of the different stocks in the market in terms of risk and return.

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CHAPTER 1: INTRODUCTION

1.1 Background to study

The connection between returns and risk is one of the fundamental issues in investment theory, and has since motivated a great deal of enquiry in both the theoretical and empirical financial economics fields for many years. This is not surprising given the fundamental role that risk plays as the major element in financial and economic decision-making. This relationship is described by Merton's (1973) capital asset pricing model (CAPM) and other asset pricing models such as the arbitrage pricing theory (APT). The CAPM envisages that the expected return on an asset above the risk-free rate is linearly related to beta, which is a measure for non-diversifiable risk, meaning the higher risk premium is expected with the increasing the beta.

Markowitz (1952) asserts that expected returns have a positive and linear connection with their expected variance, whereas Merton (1973) suggests that conditional expected returns are a positive function of their conditional variance when taking into account the degree of investor risk aversion. Moreover, Merton (1973) states that in accordance with the hypothesis of constant investment opportunity set of the Gaussian distribution of rate of returns, future returns are direct and proportionate to the product of both expected variance of returns and the measure of risk averseness. Since investors are risk averse in nature, they make an investment given that the expected returns from that particular investment are lucrative enough to reward them on the risk undertaken; hence, the risk premium is expected to have a positive sign.

The theory of Merton (1973) has been extended to include the hypothesis that investors are mean variance optimisers, meaning that they always seek the highest return with the lowest possible amount of risk. The relationship between returns and risk is often deemed a trade-off, since potential returns rise with an increase in risk. This implies that low levels of volatility bring with them low returns, whereas high levels of volatility are associated with high returns. Therefore, higher profits from investing can only be attained by those investors who are keen to take the high risk. For investors, this trade-off is a vital part of every investment decision they make.

In addition, the trade-off between risk and returns is of particular importance in asset portfolio diversification. When designing investment portfolios, asset managers decide on the weightings of diverse assets to be held in the portfolio. For a portfolio to be diversified, chosen assets must have low correlation. There is no unique formula for the optimal selection of assets, and therefore the allocation of assets depends on the investor's unique characteristics related to risk appetite, age and investment horizon (Nuttall et al., 2000). Moreover, before making an investment, investors may take into account the risk and the associated reward to determine whether it is rational to take such an action or not. For a portfolio of assets, the type of assets held and the corresponding risk levels of the asset mix are important to investment decisions.

The examples provided show how risk is vital in the determination of financial returns, and consequently this underscores the importance of good volatility forecasts in decision-making by market agents.

Ghynsels *et al.* (2005) referred to the positive trade-off between risk and returns (as suggested by theory and mainstream contributions) as “the first fundamental law in

finance”; however, empirical evidence has not always been consistent with this expectation. For instance, French *et al.* (1987) and Guo (2001) document a positive relationship, while, on the other hand, Campbell (1985), Lee and Ryu (2013), and Glosten *et al.* (1993) find evidence of a negative relationship. Expectation of relatively higher risk premiums during periods when returns from stocks are volatile is documented as one of the reasons why results are not consistent with theory. However, this is not necessarily the case, since highly volatile periods may coincide with periods when investors are better capable of tolerating certain types of risks. Furthermore, investors may want to save fairly more during periods when the future seems riskier and as a result make a high-risk premium unnecessary. In an event that all stocks are risky and risk-free investment opportunities are also available, then the price of the risky asset may go up substantially, thus reducing the risk premium. Therefore, a positive as well as a negative sign for the covariance between the conditional variance of the excess returns on stock would be in line with theory (Glosten *et al.*, 1993).

Another reason for results to deviate from expectation is wrong specification when modelling the dynamics of conditional variance that is used to measure risk. Most scholars analyse the conditional variance by employing the GARCH framework developed by Engel *et al.* (1987). However, different models are proposed by different bodies of research to describe the volatility process. For example, Ghysels *et al.* (2005) and Leon *et al.* (2007) use mixed data sampling regression (MIDAS regression), while Whitelaw (2000) and Mayfield (2004) use regime switching models. Lundblad (2007) also states that misspecification of the model cannot be the only accountable factor, but

also the low explanatory power of volatility on returns because of the small sample size. To counteract the latter issue, Bali (2008) and Engle (2010) suggest that a solution to develop methodologies on time series with small sample sizes would be to merge the temporal and the cross-sectional dimensions of investment portfolio.

Apart from model specification and sample size being responsible for conflicting results, some literature has emerged that shows that investors' degree of risk aversion follows a pro-cyclical pattern (Lettau & Ludvigson, 2003; Kim & Lee, 2008). This assertion raises a question of whether or not the risk return relationship is dependent on the economic conditions or whether or not taking into account this issue affects the relationship. Therefore, to address this issue, an empirical estimation of the results needs to be conditioned to two different regimes associated with normal financial periods and high volatility periods.

Although this relationship between volatility and market returns has received substantial attention in the literature, not much inquiry has been done regarding the effect of financial shocks on the relationship. The effects of the financial shock are likely to be revealed through deteriorating export markets, declining foreign direct investment, falling commodity prices globally and other financial inflows. These burdens lead to volatility in the macro-economy, which, consequently, causes volatility in returns and stock prices (Chinzara & Madimika, 2009). It is therefore imperative for investors to know how periods of high volatility shape the stock market returns and the volatility structure. A study, by Nezafat and Quadrini (2012), shows that financial shocks have a significant role in explaining not only business cycle fluctuations, but also the high volatility in asset prices. For instance, a higher return is expected by investors as a

result of increasing volatility during a crisis period. However, this may differ with crises; for example, a mortgage crisis brings about a decline in asset prices and market values of asset-backed securities portfolios as evidenced by the downfall of Lehman Brothers. The impact of a subprime crisis has been a central issue in financial research during and shortly after crisis periods due to its very severe effects on financial markets and the real economy all over the globe. Therefore, this necessitated more research to be conducted in an attempt to uncover the influence of the global financial crisis and recent economic events, such as quantitative easing and the political uncertainty, most importantly regarding the effects of volatility on stock returns' behaviour.

In this research paper, we demonstrate that the basic perception of a positive risk return premium does not always hold. Indeed, this study shows that the empirical association between risk and return proves that risk is not a priced factor; the risk premium is not significantly positive in the South African Stock market; and this is the case even after accounting for periods of financial turmoil. Our results confirm that the risk-return relationship is not really time dependent on the volatility regime. In general, the results show that the risk return relationship has mixed results with E-GARCH (1, 1)-M and TARARCH (1,1)-M models, giving us a negative and insignificant relationship, while GARCH(1,1)-M, on the other hand, exhibits a positively significant risk relationship.

1.2 Motivation of the study

Similar studies on the topic under consideration have mostly focused on the markets of developed countries, with very limited research focusing on emerging markets, and most importantly South Africa; however, only a few studies have been conducted on the topic in South Africa. For example, using daily data, spanning from January 1995 to

February 2009, Raputsoane (2009) investigated the risk-return relationship using GARCH-M models. The use of GARCH-M models was based on the intuition that it is ideal to model the relationship given its ability to specify the heteroskedastic conditional variance term directly into the mean equation, thereby allowing the changes in the excess returns and conditional variance to show simultaneously. Raputsoane (2009) found evidence that lends support to the existence of a positive risk premium in the South African stock market. However, he assumed that the errors are normally distributed, although it has been proven in practice that the hypothesis of normality does not always hold. In this study, we will cater for this by using different distributions of the error terms. Unlike Raputsoane (2009), Chinzara and Madimika (2009) examined the risk-return relationship by employing GARCH models under three different distributional assumptions. Their results show limited evidence to support the notion of risk being a priced factor, with some sectors even giving significant evidence of a negative risk premium in the South African stock market. The negative relationship was attributed to skewness as a factor that could have an influence on the stock returns. The authors believed that the third moment (skewness) and the fourth moment (kurtosis) should also be incorporated in the GARCH specification. In their second objective, they observed that both the political shock and the financial crises are a result of structural breaks in the trend of volatility.

This dissertation builds on the previous empirical work on South Africa by investigating the risk-return relationship in the South African stock market. This study, however, seeks to augment the information already disseminated by investigating how the recent global financial crisis and the political instability that have beset South Africa might have

impacted the relationship between the stock market returns and risk as measured by the conditional variance in South Africa. Although the risk-return relationship has been studied in the South African stock market, to the best of my knowledge, no research has been conducted to test the impact of the recent developments, such as the global financial crisis and political uncertainties, on South Africa. The regime switching GARCH-M is employed to examine the relationship between risk and returns

1.3 Objectives of the study

The objective of this study is to understand the nature of trade-off relationships between risk and returns on the JSE in periods of both temporary and persistent financial shocks. Findings should contribute to our understanding of how investors are compensated during higher or comparatively lower risk periods. To address this objective, the study will:

- Investigate the effect of a financial shock on the risk-return trade-off when accounting for asymmetric volatility phenomenon, using GARCH-M as well as its variants, E-GARCH-M and TARCH-M models.
- Estimate all the above-mentioned models under the three error distributions, explicitly the normal distribution, student-t distribution and the generalised error distribution (GED).
- Compare the estimated models to determine which one is the best mostly for modelling volatility.

1.4 Importance of the study

The risk-return relationship has become a fundamental concept in financial markets. If financial decision making is meant to lead to benefit maximisation, it is vital that individuals consider the combined influence on expected returns as well as risk. South Africa is the largest stock market in Africa; however studies on the topic are still limited. It is very crucial for the relationship to be understood by investors as well as managers as the financial markets contribute a substantial amount toward the country's GDP. This study therefore serves as an instrument for researchers as well as scholars to understand the relationship of risk and returns in financial instruments and how the relationship is affected by regime changes. This study will be of reference to analysts, giving them better insight when testing the behaviour of the South African stock market thus help in informed decision making to avoid risk commonly inherent in the investment industry. Not only is this study useful in providing insight to researchers, it is also aimed at providing a better understanding to investors when making investment decisions. If investors have an understanding of how their markets behave under bullish or extremely volatile periods they will take into consideration risk and different volatility regimes before making any investment decision.

1.5 Organisation of the study

The remainder of the dissertation is organised as follows: The chapter that follows reviews both theoretical and empirical literature relevant to the risk-return relationship both within the framework of the South African stock market and international studies. This chapter is divided into two sections: The first section outlines the theoretical framework, informing us on the *a priori* expectation of the risk-return relationship,

specifically the capital asset pricing model (CAPM) and the arbitrage pricing theory (APT), and the second section covers the empirical literature. Chapter 3 presents the methodology used in this study. The results of this study are presented and analysed in Chapter 4. Conclusions, policy recommendations and areas of further research are discussed in the last chapter.



CHAPTER 2: LITERATURE REVIEW

2.1 Introduction

This section presents a review of the risk-return literature on the stock market which is one of the most imperative theories in financial investment. Risk is proxied by volatility as it measures deviations from expectation and, in this regard, risk is the negative deviation from the expected value of stock prices. This relationship is behind the theoretical foundation of numerous investment theories, such as CAPM and APT. Although many studies have been piloted regarding the relationship between risk and returns for financial markets of developed countries, limited research has been conducted in emerging markets, especially those in Africa. Given the recent global financial crisis and the recent political uncertainties in South Africa, it is imperative to study this relationship and to analyse how the relationship is affected by different volatility regimes. Moreover, the choice of South Africa is not arbitrary, but very interesting. South Africa is an emerging economy but boasts a very strong financial sector that had assets of over R7 trillion in 2017, contributing at least 10 percent of GDP and is on course to be Africa's financial hub.

The first section of the literature review discusses the theoretical literature, where the two asset pricing models CAPM and APT will be outlined. This is followed by the critical analysis of empirical literature, which will be presented in subsection 2 of the chapter, and the last section of the chapter will conclude.

2.2 Theoretical literature review

This section discusses in detail the two theories explaining the link between risk and return, namely CAPM by Sharpe (1964) and Treynor (1961), which was later developed by Litner (1965) and Mossin (1966). An alternative equilibrium asset pricing model, APT by Ross (1976), was also discussed in the study. CAPM emanates from the model of portfolio theory by Markowitz (1952), while APT is an extension of CAPM. Both the CAPM and APT are mechanisms used for the prediction of financial asset prices, and are therefore of significance in helping investors channel savings into profitable investments, and consequently aid in the optimal allocation of capital.

2.2.1 Capital asset pricing model

The CAPM is one of the fundamental models in financial economics as it is used as a benchmark model for pricing securities. The CAPM was developed by Sharpe (1964) and Treynor (1961). It proposes that, the market portfolio is the most diversified portfolio relative to other portfolios; as a result the risk premium of a stock should be proportional to the expected risk premium of the market portfolio.

According to Bailey (2005), the predictions of the CAPM are built on the following assumptions:

1. Stock markets are in equilibrium; there are no frictions, i.e. the market and prices adjust such that stocks of assets are held voluntarily.
2. Investors behaviour is based on a mean-variance objective.
3. Investors have homogenous beliefs; they analyse and view securities in the same economic view of the world.

CAPM model is usually expressed by the following equation:

$$E(R) - R_F = \alpha_i + \beta_F (E(R_m) - R_F) + \varepsilon_{it} \quad 2.1$$

$$E(R) = R_F + \beta_F (E(R_m) - R_F) + \varepsilon_{it} \quad 2.2$$

where R , R_F and R_m are the returns of a financial asset, risk-free rate and the return of the market portfolio, respectively. R and R_m are random variables.

The left side of equation 2.1, $E(R) - R_F$, represents the risk premium of an asset, while $(E(R_m) - R_F)$ is the risk premium of the market portfolio. Alpha (α_i) represents the risk-adjusted measure of rate of return on an investment, i.e. the return independent of risk. Beta (β_F) is the slope coefficient that shows the responsiveness of asset movements in the market. The value of β_F greater than 1 is an indication that the asset is riskier relative to the market. The error term ε_{it} is the idiosyncratic risk of the asset. Given different assets with negative correlation, investors' exposure to idiosyncratic risk is avoidable and very small, and uncorrelated with the rest of the portfolio. In a fully diversified portfolio, $E(\varepsilon_{it}) = 0$, which means that idiosyncratic risk is 0. Therefore, the distribution of idiosyncratic risk to the risk of the whole portfolio is insignificant (Javed, 2002).

The beta parameter is very important in the CAPM, as it measures systematic risk. The coefficient is represented as follows:

$$\beta_F = \frac{Cov(R_{it}, R_{mt})}{Var(R_{mt})} \quad 2.3$$

where $Cov(R_{it}, R_{mt})$ is a measure of how changes in an individual stock's returns relate to changes in the overall market's return and $Var(R_{mt})$ is the variance of the entire market (Fama & French, 2004).

The CAPM categorised risk into two components; the first element is systematic risk, which is non-diversifiable, and the second is idiosyncratic risk, which can be diversified away, and consequently investors are not rewarded for taking this type of risk. Therefore, based on CAPM, the risk associated with returns should be attributed to systematic risk. Basically, the CAPM postulates that for all risk-averse investor, there is an equilibrium relationship between risk and the expected return for every asset. When there is market equilibrium, an asset will provide an expected return proportionate to its systematic risk. The more the systematic risk of an asset, the greater the return that investors will expect from it.

2.2.2 Arbitrage pricing theory

APT was established by Ross in 1976, as an alternative to CAPM. It is considered an alternative in that they both emphasize on a linear relation between the stocks' expected return and their covariance with other variables. However, the APT differs from the CAPM where it assumes that prices are affected by both firm-level and macro-innovations. APT is not only focused on assessing the performance of the market, it also associates stock price to the dynamics affecting it. The issue with this is that the theory in itself has no suggestion of what these dynamics are, so they have to be empirically determined. The potentially large number of factors means more betas to be calculated and yet there is still no assurance that all the important factors have been identified (Sharpe, 1992). Therefore, the APT is practically difficult to implement.

According to the APT, investors are rewarded for all factors that have an impact on the return of a security. The reward is measured as a sum of the factors of each risk factor's systematic risk and the risk premium assigned to it by the capital market. Supporters of the APT model claim that it has the following benefits over the CAPM:

1. The APT has fewer limiting suppositions regarding investor preference towards risk and return.
2. There are no assumptions regarding the distribution of asset returns.
3. The model assumes that the only way an investor can increase the returns of their portfolio is by increasing wealth and risk.

The APT model is the most appropriate an asset pricing model where there is more than one risk factor in a multifactor risk model (Focardi *et al.*, 2004).

2.3 Review of the empirical literature

2.3.1 Risk-return behaviour in the South African stock markets

Raputsoane (2009) examined the intertemporal risk-return relationship. In this study, the GARCH-M model is employed to analyse the market and stock price index returns of selected listed companies from the Johannesburg Stock Exchange for the period 1999 to 2009. The author observed that 95% of securities indices presented a positive and statistically significant coefficient of risk aversion, and therefore the South African stock market supports the positive risk-return relationship hypothesis.

Another study carried out on the South African framework is where Madimika and Chinzara (2010) analysed the risk-return relationship, the behaviour of volatility and the long-term trend of volatility on the South African stock market. Using sectoral-level,

industrial-level and aggregate-level daily data, covering the period 30/06/1995 to 31/07/2009, dummy variables were assigned for the Asian, the subprime crises and the September 11th political shock. The three GARCH models were then estimated under the three distributional assumptions, explicitly the normal distribution, student-t distribution and the generalised error distribution. Contrary to Raputsoane (2009), the observed results indicate that at industry and sector levels, volatility is generally persistent, and there is significant indication of leverage effects and symmetry. However, for a handful of sectors, volatility was found not to be priced, as the risk premium was observed to be negative in some sectors, while in some it was found to be positive, but insignificant. Although GARCH models were used by Raputsoane (2009) and Madimika and Chinzara (2010), differences were observed in their results. These differences may be a result of differences in the frequency of data used: Madimika and Chinzara (2010) used daily data, whereas Raputsoane (2009) used monthly data. Previous studies on the South African stock market have proved that the South African stock market is informationally efficient (Mkhize & Msweli-Mbanga, 2006). Therefore, higher frequency (daily) data will deliver better dynamics of the return generating process as opposed to lower frequency (weekly) data. Raputsoane's (2009) results are in support of the CAPM hypothesis, which states that investors are expected to be remunerated for risk taken and the time value of money.

Using data for the total stock return index in South Africa for the period January 1973 to December 2011, Darrat *et al.* (2012) also tested the risk-return relationship in the South African stock market. The GARCH models were estimated for three data frequencies (weekly, monthly and quarterly). They found that the results are against the

hypothesised positive relationship between risk and return in South Africa. Darrat *et al.* (2012) also continue to examine the relationship using the plain vanilla time series model, similar to their GARCH model's estimations, and they still observe that the positive risk-return relationship postulated by the CAPM is not supported. This observation is, however, different from Raputsoane's, even when the comparison of results is conducted on monthly data model estimates. The differences in results could also be perceived as a result of different data periods used. Raputsoane (2009) used data from 1999 as opposed to Darrat *et al.* (2012), who used data that dated as far back as 1973.

2.3.2 Risk-return literature on the international stock markets

In their study, French *et al.* (1987) test whether the risk premium is positive in the stock market. They use two different statistical methods to investigate the relation by using the S&P composite portfolio's daily returns from the period January to December of 1984. The first method decomposed the estimates into both the predictable and the unpredictable components with the use of autoregressive integrated moving average (ARIMA) model. They found little evidence supporting the positive relationship; however, the strong negative relation was observed for the unpredictable component and therefore these observations are interpreted by the authors as in indirect support of a positive relationship. The GARCH-M model is also used in the estimation of the *ex-ante* relationship between volatility and risk. These results are also in support of the ARIMA models that advocate a positive relationship between expected risk and volatility.

Glosten *et al.* (1993) also study the relationship between the volatility and the nominal excess returns on stock by using the specification of the GARCH-M that incorporates dummy variables designed to account for different volatility regimes. The model they use also captures a leverage effect which is a characteristic mostly observed in financial data. The authors observed a weak but statistically significant relationship between volatility and expected returns.

Theodossiou and Lee (1995) tested the link between the risk and expected return employing a GARCH-M model. The aim of their study was to discover the nature of the stock market risk and how it relates with expected return for the following ten industrialised markets: West Germany, the United States, the United Kingdom, Switzerland, Japan, Italy, France, Canada, Belgium and Australia. No apparent relationship between the two variables of interest was observed in any of the ten markets.

Margot and Whyte (1996) studied the relationship in the German and French stock markets. They found volatility to have a trivial effect on stocks. The data used in the study was daily stock return data for both aforementioned markets spanning from 31 December 1979 to 7 July 1991. The models employed in the study were GARCH models, under the student-t distribution assumption. Furthermore, this relationship was tested taking into account the 1987 stock market crash to see whether it had any effect on the relationship in the two countries. This consideration indeed improved the model's fit insignificantly. The risk aversion parameter was found to be positive for the two countries under consideration, but only found to be significant in Germany. The main

finding of this study suggests that taking into account structural shifts is robust when determining the relationship between stock returns and volatility.

Using the GARCH-M model, Poshokwale and Murinde (2001) analysed the risk return relationship on the Hungarian and Polish daily stock market prices. The study covered the period from 1994 to 1996. Contrary to the predictions of the CAPM that suggest high volatility leads to increased expected returns, the results indicated that returns showed that the risk premium was not positive in both markets. Similar findings were observed by Yu and Hassan (2008), using the E-GARCH model that allows for asymmetries in financial data. Using daily data from the seven countries from the Middle Eastern and North African (MENA) stock markets, they observed a positive significant positive risk-returns relation, which is in support of the CAPM hypothesis in the three countries explicitly Bahrain, Oman and Saudi Arabia. However, in the four remaining countries namely Egypt, Jordan, Morocco and Turkey, the risk premium was not found to be positive. Karmakar (2007) employed the same model on the Indian stock market; nonetheless, he also found a negative relationship on the S&P CNX Nifty for the period spanning from 1990 to 2004. Similarly, Saleem (2007) used the EGARCH-M model to also test the relationship focusing on the period between 1997 and 2004. The author observed that positive returns corresponded with higher volatility for Pakistan's daily stock of the KSE-100 index.

Guo (2002) investigates the link between previous stock market variance and its future excess market returns. In order to achieve this, they estimate Merton's intertemporal capital asset pricing model (ICAPM). Following Pagan (1984), the author estimates the ICAPM equation and the conditional variance equation jointly using two-stage least

square regressions. They found that risk is a positively priced factor; that is, past market variance has a positive correlation with future excess returns and market stock variance. However, their results show that, when the consumption wealth ratio is used as an instrumental variable, then conflicting results are observed as opposed to when implied volatility is used.

Muller *et al.* (2010) investigate Merton's hypothesis on the risk-return trade-off. They employed the combination of both traditional discrete time GARCH models and the continuous time generalised orthogonal GARCH (O-GARCH) model, with the latter enabling them to analyse unequally spaced data. Daily values for weighted indices from the Centre for Research in Security Prices covering the period 1953 to 2009 were used. A period of this length was used in order to allow for all the variations in expected returns to be covered. Daily data was preferred because it is believed to give allowance for the fine structure of volatility to be considered. The authors employed the GO-GARCH model, as it is proficient when daily data is used, as it solves the problems related to the use of daily data, such as Friday effects and the effects of data discontinuities that occur as a result of weekends and holidays when trading does not take place. They found that a risk premium of 7 percent per annum exists when a risk return relationship is symmetric to positive or negative returns. However, they observed that the inclusion of the asymmetry effect produces an insignificant risk premium of 7 percent per annum. In an attempt to test the relationship between the expected returns expected returns and volatility in the Chinese stock market, Xiao and Zhao (2013) incorporated the multivariate GARCH-M model with dynamic conditional variance

(DCC). Their results lend support to the existence and validity of conditional CAPM hypothesis.

Sekmen and Hatipoglu (2015) used the recent crisis as a special reference period to examine how the risk-return relationship responds to different volatile regimes on the Turkish stock market. Data was divided into three periods centred on the 2008 financial crises. The first period which was labelled the pre-crises period started on the 4 June and ended on the 29 July 2007; the second period spanning from 2 June 2007 to 2 April 2009 was the crises period, and the last period which was post-crisis period, started on the 3 April 2009 and ended on 3 June 2014. Their findings revealed that the crisis had a positive and temporary effect on the volatility of returns in the stock market. Moreover, the authors also observed that the crisis induced a fairly significant increase in the asymmetric parameter, which shows that negative market news have a superior effect on future volatility relative to positive news.

Arewa and Ogbulu (2015) investigated whether the priced and non-priced risk factors have an impact on stock returns in the Nigerian stock market. In order to accomplish their objective, they based their theoretical analysis on the CAPM with the higher order. It is observed that the CAPM intercept is not statistically different from zero, and therefore there is support of the CAPM hypothesis. Furthermore, although not significant, the positive sign of the slope coefficient that is observed is also in support of CAPM's positive-risk return hypothesis. Arewa and Ogbulu (2015) also observed systematic co-skewness risk as the only risk that is significantly priced when applying the unconditional four-moment CAPM, thereby showing that Nigerian capital markets pay premiums to investors to take risk. Further tests on the relationship were conducted

by Hongsakulvasu (2015), who employed a semi-parametric GARCH mean model in an attempt to address the problems that may be attributed to the misspecification of either the mean specification or the conditional variance equation, or both. The results indicate that risk has a very negligible effect on the returns of the S&P 500, and therefore it has little effect on the market return. However, it was further observed that adding four variables to the models provides different results that show the risk-return relationship to be positive.

2.4 Conclusion

Theoretical models propose that there is a positive relationship between risk and return (the arbitrage pricing theory and the capital asset pricing model). However, an empirical studies review is not very conclusive on the relationship. In some instances, there is evidence that it does not always follow that assets with high risk will have high returns, while, in other cases, there is the existence of a positive, although statistically insignificant, relationship between systematic risk and returns.

Campbell (1987) and Scruggs (1998) pointed out that there are two aspects that are accountable to the failure of research to attain a reasonable conclusion on the risk-return relationship. Firstly, the major obstacle when estimating the coefficient of relative risk aversion in GARCH models is that conditional market variance is not directly observable and certain constraints should be enforced in order to detect it from past returns; and secondly, model misspecification could also pose issues in obtaining the positive risk-return relationship.

Another reason for the different results might stem from the use of different empirical methods in the modelling of risk, such as autoregressive integrated moving average (ARIMA) by Jenkins (1976), which was used to assess the volatility of financial assets, exponentially weighted moving average to measure volatility, and also the Black Scholes asset pricing models, which were used to determine implied volatility. These models were built on the assumption of constant variance over the period of the financial series of interest. As a result, this became a limiting factor as they failed to account for stylised facts about volatility, such as leptokurtosis, volatility clustering, non-normality, fat tails, the leverage effect etc. Engle's ARCH models and its extensions gained popularity because of their ability to account for such stylised facts. Some of the key issues faced by researchers is the lack of concise way to measure risk. Concerns have been raised on whether Gaussian-type GARCH model is an appropriate procedure to derive conditional variance to test the risk return relationship. Some researchers like Bali et al. (2009) and Chen and Chiang (2016) preferred VaR for applying the downside risk analysis when market encounters extreme shocks as the conditional variance cannot sufficiently model the risk averse behaviour. In this paper I argue that TARCH-M and E-GARCH models are sufficient in estimating the relationship as they account for time-Varying nature of volatility and the stylized facts on financial data. Not only do they account for the stylized facts on financial data they also account for the leverage effects which is not accounted for by other models.

Firstly, it will be interesting to know the nature of this relationship within the South African context; that is, whether the relationship conforms to the theoretical expectation

or otherwise, and the reasons for non-conformity discussed. This would surely add to the knowledge on the nature of the risk-return relationship. Lastly, no study has been undertaken to examine the behaviour of this relationship in periods of different volatile regimes. As mentioned earlier in the review, it was observed that the Turkish market responds differently to different volatile regimes (Sekmen & Hatipoglu, 2015). Knowledge of the risk-returns behaviour in different regimes of the South African market would not only add to existing knowledge, but would also inform investors and policymakers. Moreover there has been no study carried out on the effect of financial crises on the risk return relationship in South Africa.



CHAPTER 3: METHODOLOGY AND ANALYTICAL FRAMEWORK

3.1 Introduction

The purpose of this chapter is to present the analytical framework employed to understand the nature of the trade-off between risk and returns on the JSE during normal financial periods and periods of financial shocks. Financial theory states that there is a potential increase in returns when risk increases, and therefore the relationship is linear. In the subsequent sections, the dissertation discusses the time series techniques employed to capture the stock returns as well as volatility estimation models.

3.2 Time series techniques to capture stock return dynamics

Under the theory of financial returns, it is a basic intuition that returns follow a time series model (Brooks, 2008). Analysis of time series data takes into account the fact that data points taken over a period of time might have an internal structure (such as autocorrelation, trend or seasonal variation) that ought to be taken into consideration when modelling such data.

Given a stationary time series x_t , its evolution can be explained by the following process:

$$x_t = E(x_t / \Omega_{t-1}) + \varepsilon_t \quad 3.1$$

where $E(x_t / \Omega_{t-1})$ represents the conditional expectation operator, Ω_{t-1} denotes information set at time $t - 1$ and ε_t represents innovations of the time series. According

to Asteriou and Hall (2007), the techniques developed in the time series forecasting have become increasingly useful in the estimation of financial data, and consequently the section that follows will briefly discuss the univariate time series models applied in the study.

3.2.1 Moving average process

A moving average process is a linear combination of the white noise processes; such that x_t is determined by the current and previous values of a white noise disturbance term (Brooks, 2008). The model representation is as follows:

$$x_t = \sum_j^q b_j e_{t-j} + e_t \quad 3.2$$

In the above MA (q) process, the q symbolizes the order as well as explains the number of time lags the model takes into account. The term e_t represents random shocks, i.e a random noise for which;

$$E(e_t) = 0. \quad 3.3$$

$$Var(e_t) = \sigma^2$$

where σ_t^2 represents conditional variance of the innovation. The MA (q) process is said to be stationary if the process is invertible. An MA (q) process is invertible if it can be rewritten as an AR (p) process. For this to happen, the roots of the MA (q) process should be more in absolute values (Anderson *et al.*, 2012). Below is the mathematical representation of the MA (q) process:

$$1 - b_1 x - b_2 x^2 - b_3 x^q = 0 \quad 3.4$$

3.2.2 Autoregressive process

An autoregressive process is a linear combination of the previous values of a time series, i.e. time series x_t is dependent on its previous value x_{t-1} plus an error term. The mathematical representation of an autoregressive model of order p denoted by AR (p) takes the following form:

$$x_t = u + \alpha_1 x_{t-1} + \alpha_2 x_{t-2} + \dots + \alpha_p x_{t-p} + e_t \quad 3.5$$

This can be rewritten as

$$x_t = u + \sum_{i=1}^p \alpha_i x_{t-i} + e_t \quad 3.6$$

Likewise, in the AR (p) process, p represents the order and also explains the number of time lags the model considers. These orders also represent the number of parameters in the AR (p) process. In equation 3.7, e_t represents random innovations for which:

$$E(e_t) = 0 \quad 3.7$$

$$Var(e_t) = \sigma^2 \quad 3.8$$

Just as with the MA (q) process, the condition that the roots of the characteristic model should be greater than one in absolute terms applies to the general AR (p) process (Anderson *et al.*, 2012).

The mathematical representation of the AR (p) process is given by:

$$1 - \alpha_1 x - \alpha_2 x^2 - \alpha_3 x^3 - \dots - \alpha_p x^p = 0 \quad 3.9$$

3.2.3 Autoregressive moving average

The autoregressive moving average models are a class of univariate time series models used for short-term forecasting of the second-order stationary stochastic process, which

provides a benchmark for structural models. It is a combination of both the autoregressive and moving average models, which results in a new series, the ARMA (p, q) model. In essence, the statistical implication of this model is that some series y linearly depends on its own past values together with both the current and past values of the white noise error term. According to Brooks (2008), ARMA models are not constructed on the basis of any underlying theory about the behaviour of the variable; rather, it only captures significant features of the observed data. The general form of the ARMA model is an ARMA (p, q) model, which is presented as follows:

$$x_t = \alpha_1 x_{t-1} + \alpha_2 x_{t-2} + \dots + \alpha_p x_{t-p} + u + b_1 e_{t-1} + b_2 e_{t-2} + \dots + b_q e_{t-q} \quad 3.10$$

It can be rewritten as:

$$x_t = \sum_{i=1}^p \alpha_i x_{t-i} + \sum_{j=1}^q b_j e_{t-j} + u \quad 3.11$$

where u is the mean, α_i is the autoregressive coefficient and b_j is the moving average coefficient. For stationarity, invertibility for both the MA (q) part and AR (p) is required. These models are important in predicting the means included in the volatility models.

3.3 Volatility estimations

Volatility of asset price returns is characterised by large departures from their expected levels, and therefore variance that measures the variability from an average is considered a good proxy for volatility. In the case of our study, this measure is important as it determines by how much the stock prices deviate from the mean during different financial periods. Volatility of financial time series is widely known to be time varying in nature, and therefore there is a need for appropriate models that will capture this time varying nature. A good volatility model should be capable of accounting for empirical

regularities found in financial data. Generally, it is a widely observed phenomenon that financial time series are characterised by heteroskedastic variances of the error term. As a result of the heteroskedastic nature of the variance of the error term, Engle (1982) introduced the ARCH technique to model the heteroskedastic variance of a time series data applied to an ARMA process. The variance of the innovations in the time series process is now being generated from the ARCH process. The ARCH process was defined by Engle (1982), where all error terms (ε_t) take the form:

$$\varepsilon_t = z_t \sigma_t \quad 3.12$$

where z_t is an i.i.d. process with zero mean and variance one and σ_t is conditional standard deviation.

The following section will delve into ARCH models, GARCH models and their variations.

3.3.1 ARCH: Autoregressive conditional heteroskedasticity

ARCH models are very significant in capturing volatility in asset returns. The ARCH model was introduced by Engle (1982) in his seminal work to model the time varying nature of conditional variance. These models were constructed in such a manner that they captured volatility clustering and leptokurtosis, which are some of the stylised facts commonly observed in financial data (Mandelbrot, 2002). The first assumption, as explained by Brooks (2008), is a trend for volatility to happen in clusters, implying that periods of high asset returns are typically followed by periods high assets returns, and in the same manner, periods of small asset returns are followed by periods of small asset returns. Secondly leptokurtic distributions have heavier tails and are heavily

clustered around the mean value. The ARCH model estimates the variance of returns as a function of lagged value of innovations, as presented below:

Given x_t as a time series of financial asset returns whose mean equation is given by ε_t

$$x_t = E(x_t / I_{t-1}) + \varepsilon_t \quad 3.13$$

where I_{t-1} represents the information set at time $t - 1$ and ε_t are random innovations with $E(\varepsilon_t) = 0$

Then, the variance equation is presented as:

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2 \quad 3.14$$

where $\varepsilon_t = \sigma_t z_t$, and z_t denotes an i.i.d Gaussian process with mean zero and variance one.

3.3.2 Generalised autoregressive conditional heteroskedasticity: GARCH

This section explains the GARCH model, which is the generalisation of the ordinary ARCH model. The model structure was developed by both Taylor (1986) and Bollerslev (1986) independently. These developments on the ARCH estimation techniques models were due to limitations that the model possessed. According to Bollerslev (1986), one of the limitations of the model is that ARCH needs to be of a high lag order to be able to sufficiently encapsulate the dynamic behaviour of volatility. It is these limitations that led to the need to improve the ARCH model, and this led to the advancement of the GARCH model, which was intended to overcome this issue. In the basic GARCH (p, q) model, variance σ^2 at time t is expressed as:

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2 + \beta_i \sigma_{t-1}^2 \quad 3.15$$

where σ_t^2 is the conditional variance and $\alpha_0, \alpha_i, \beta_i$, are parameters to be estimated.

ε_{t-i}^2 is the square of previous errors, which handles the effects of innovations and is referred to as the ARCH term.

σ_{t-j}^2 is predicted previous conditional variance, and reflects the impact of past volatility on current level of volatility.

Current volatility (σ_t^2) depends on p lags of past conditional volatility σ_{t-j}^2 and square of previous errors (Brooks, 2008). There are a number of constraints imposed on the model. Firstly, $\beta_i \geq 0$ and $\alpha_i \geq 0$ are necessary to confirm that σ_t^2 is not negative, and secondly, $\alpha_i + \beta_i < 1$. The latter constraint ensures that the unconditional variance of ε_t is constant, even though its conditional variance changes overtime. The unconditional variance is given by $Var(\varepsilon_t) = \frac{\alpha_0}{1-(\alpha_i + \beta_i)}$ when $\alpha_i + \beta_i < 1$ and $\alpha_i + \beta_i$ is also a measure of persistence. For $\alpha_i > 1$, the unconditional variance of ε_t is undefined, and the model is non-stationary in variance. According to Brooks (2008), conditional variance forecasts of stationary GARCH models converge upon the long-term mean of the variance as the time horizon for forecast increases. Conditional variance is given by a combination of both the unconditional variance and the deviation of squared error from its expected value.

The GARCH model makes it possible to understand the variance as a weighted function of long-term expected value (depends on α_0), information about volatility in the prior period ($\alpha_1 \mu_{t-1}^2$) and the fitted variance from the model during the prior period ($\beta_0 \sigma_{t-1}^2$). Relative to the ARCH model, the GARCH model is an attractive choice as it uses the least assumptions and variables, whilst avoiding over-fitting (Brooks, 2008).

A basic ARCH model is a unique case of GARCH specification, where lagged forecast variance does not exist; that is, GARCH (0, q) and $\alpha_1 = \alpha_2 = \alpha_3 = \dots = \alpha_p = 0$

Although GARCH models with conditional normal distribution permit unconditional error distribution to be leptokurtic, they may not adequately explain the high level of kurtosis in distribution of return series (Bakry, 2006). Literature suggests that the assumption of a leptokurtic conditional distribution in GARCH models might be more suitable, and therefore the student-t and the generalised error distributions are necessary to allow for this type of distribution. These two distributions are better able to explain the level of kurtosis found in financial data than the normal distribution can.

3.3.3 Exponential GARCH (EGARCH)

This model was advanced by Nelson (1991) to overcome the GARCH model's limitation of not taking into consideration the leverage effect. The EGARCH, unlike the ARCH and the GARCH models that suggest adverse and constructive shocks to have the same effect on the forecasted volatility, is able to model good news and bad news differently, whereas, with GARCH models only the size, not the sign of returns matters in determining volatility. On the left-hand side of the variance equation is the log of the variance of the series which ensures non-negativity in the estimates of time-varying variance. The variance equation for the EGARCH is represented as follows:

$$\text{Log}(\sigma_t^2) = \alpha_0 + \sum_{j=1}^q \alpha_j \left| \frac{\varepsilon_{t-1}}{\sqrt{\sigma_{t-1}^2}} \right| + \sum_{j=1}^q \gamma_j \frac{\varepsilon_{t-1}}{\sqrt{\sigma_{t-1}^2}} + \sum_{i=1}^p \beta_i \log(\sigma_{t-1}^2) \quad 3.16$$

The parameters to be estimated are $\beta_i, \delta_j, \gamma, \alpha_i$. However, the most important parameter is γ_j as it allows for the asymmetric effects to be captured. For example, if $\gamma_j = 0$, it is a suggestion that the model is symmetric, and if $\gamma_j < 0$, this means that positive shocks,

i.e. good news, cause less volatility relative to the negative shocks (bad news) do. In financial time series data, negative shocks have been found to cause volatility to increase by more than a positive shock of the same magnitude. These asymmetries are usually attributed to a number of reasons; the first being the leverage effect, in which the firm's leverage ratio increases because of the deteriorating value of a firm's stock, thereby leading to shareholders bearing the residual risk of the firm. The second reason is the volatility feedback theory. Under the assumption of constant dividends, if expected returns and stock price volatility increase simultaneously, then stock prices should fall when volatility rises (Brooks, 2008).

Another limitation of GARCH models that EGARCH addresses is the non-negativity constraint imposed on γ to make sure that σ_t^2 remains positive for all t . The implication of these constraints is that an increase in returns in any period increases σ_{t+m}^2 for $m \geq 1$. The left-hand side of the EGARCH model is the log of the variances of the series. These logs make the leverage effect exponential, and therefore lead to guaranteed positive forecasts of the conditional variance, without artificially imposing a non-negative constraint on the parameters. The EGARCH model provides an explanation for the leverage effects. Furthermore, the model also explains the size of the lagged residuals and why σ_t^2 is dependent on both the signs.

3.3.4 Glosten-Jagannathan-Runkle GARCH (GJR-GARCH)/Threshold GARCH

The GJR GARCH model, also known as the TARARCH model, is one more extension of the GARCH model that was developed by Glosten, Jagannathan and Runkle in 1993 and Zakonian in 1994. The TARARCH models were developed to better capture the movements of negative shocks because of the bigger effect they impose on volatility

(Tsay, 2005). According to Brooks (2008), the basis for the TARCH is like to the EGARCH model as negative news has a greater influence on volatility than good news of the similar magnitude does. This model introduces a threshold effect in the form of a dummy variable into volatility to account for leverage effects. The specification of the conditional variance equation for the TARCH (1, 1) is presented as follows:

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2 + \gamma \varepsilon_{t-1}^2 I_{t-1} \quad 3.17$$

$$\text{with restrictions } I_{t-1} = 1 \text{ if } \varepsilon_{t-1}^2 < 0 \quad 3.18$$

$$= 0 \text{ otherwise}$$

A leverage effect exists if $\gamma > 0$, and the non-negativity condition is given by $\alpha_0 > 0$, $\alpha_1 > 0$, $\beta \geq 0$, and $\alpha_1 + \gamma \geq 0$, and for this reason, the model is still valid if $\gamma < 0$ as long as it meets the condition $\alpha_1 + \gamma \geq 0$. Given that coefficient γ is positive and robust, then the negative shocks will have a larger impact on σ_t^2 than positive shocks will. The model is stationary, and $\gamma < 2(1 - \alpha_1 - \beta_1)$, which is also used as measure of persistence.

3.4 Modelling the risk-return relationship

Among the GARCH extensions is the GARCH-M model, which is particularly important for the objective of this study. The motivation behind this specific model is to shed light on the excessive risk premium in the financial market. According to Engle *et al.* (1987), risk-averse economic agents require compensation that varies with degree of uncertainty that changes overtime. Unfortunately, traditional GARCH model do not explain this excessive return as their conditional expectation remains zero throughout the time. However, the GARCH-M model, as suggested by Engle *et al.* (1987), excels in

addressing this issue by directly establishing a risk-return relationship where the time-varying premium is a linear function of risk. The model is presented as follows:

$$r_t = \omega + \varphi \sigma_t + \mu_t, \mu_t \sim N(0, \sigma_t^2) \quad 3.19$$

$$\sigma_t^2 = \alpha_0 + \alpha_i \varepsilon_{t-1}^2 + \beta_j \sigma_{t-1}^2 \quad 3.20$$

where $\alpha_0 > 0$, $\alpha_i > 0$, $\beta_i \geq 0$, ω and φ are constants. The process r_t is the GARCH-M process of order p and q denoted by $r_t \sim GARCH - M(p, q)$.

Mostly important to this study is the coefficient of σ_t (φ) in the mean equation, since it shows the relation between conditional risk and returns. If α is statistically significant and positive, then, according to investment theory, investors are awarded for the risk taken, i.e. more risk, given by an increase in the conditional variance, will lead to an increase in the mean return. Therefore, φ can be interpreted as a risk premium.

Because of its ability to allow for the conditional mean to depend on its own conditional variance, the GARCH-M model is mostly important for the study at hand relative to other GARCH models, since it enables us to investigate the risk-return relation. However, because of the disadvantages associated with the normal GARCH (p, q) variance specification mentioned earlier, it is necessary to extend the GARCH-M model with a component to account for asymmetry, and therefore the E-GARCH-M and TARCH-M models will also be estimated in the study. We must take note that the conditional volatility models of these two models are similar to those of E-GARCH (equation 3.17) and TARCH (equation 3.18) models, respectively, while their mean equations are similar to that of GARCH-M model as presented in equation 3.20.

3.5 Choice of error distributions and their implication for forecasting

In the ARCH model, the error term is presumed to have a normal distribution; however, to better the model excess kurtosis that is prevalent in financial series, the normal distribution assumption is relaxed in this study. This dissertation explores whether the returns are better described by other distributions, namely the student t-distribution and the GED.

3.5.1 The normal distribution

The log likelihood function distribution under the normal distribution is represented as follows:

$$L(\theta_t) = \frac{1}{2} \log(2\pi) - \frac{1}{2} \log \sigma_t^2 - \frac{1}{2} (r_t - \theta r_{t-1})^2 / \sigma_t^2 \quad 3.21$$

where r_t and r_{t-1} represent present and past returns respectively, $0 < \theta < 1$ and t represents the total number of observations.

3.5.2 The student-t distribution

One of the common features of financial data is that it has fat tails, and this element is not accounted for by the normal distribution. The student-t distribution and the GED are usually used to give a rationale for this singularity. In the case of the student-t distribution, the log likelihood is specified as follows:

$$L(\theta_t) = -\frac{1}{2} \log \left(\frac{\pi(v)\Gamma(v/2)^2}{\Gamma((v+1)/2)^2} \right) - \frac{1}{2} \log \sigma_t^2 - \frac{(v+1)}{2} \log \left(1 + \frac{(r_t - \theta r_{t-1})^2}{\sigma_t^2 (v-2)} \right) \quad 3.22$$

In the above equation, $\Gamma(\cdot)$ is the gamma function and $v > 2$ represents the degree of freedom and controls the tail behaviour. Equally important, as $v \rightarrow \infty$ the student - t distribution will converge to the normal distribution.

3.5.3 The generalised error distribution

The assumption that GARCH models follow GED accounts for the kurtosis in returns, which are not adequately captured under the normality assumption. The log likelihood function under the GED, as proposed by Nelson (1991), takes the following form:

$$L(\theta_t) = -\frac{1}{2} \log \left(\frac{\Gamma(\frac{1}{v})^3}{\Gamma(\frac{3}{v})(\frac{v}{2})^2} \right) - \frac{1}{2} \log \sigma_t^2 - \left(\frac{\Gamma(\frac{3}{v})(r_t - \theta r_{t-1})^2}{\sigma_t^2 \Gamma(\frac{1}{v})} \right)^{v/2} \quad 3.23$$

v is the shape parameter that accounts for the skewness of the returns and $v > 0$. The weight of the tail will be greater as the value of v increases. GED reverts to normal distribution if $v = 2$, and fat tailed if $v < 2$.

3.6 Conclusion

This chapter explained the methodological framework employed in the study to test the risk-return relationship in the South African stock market. ARMA models, then univariate GARCH models along with asymmetric GARCH models, namely E-GARCH and TARCH models, as well as a family of GARCH-M, were all explained. The three error distributional assumptions under which the GARCH models were estimated were also explained. The chapter that follows will present the data as well as results and interpretations.

CHAPTER 4: EMPIRICAL RESULTS

4.1 Introduction

This chapter presents the results from the estimations carried out as proposed in the methodology as well as interpretations to the observed results. The first section of the chapter entails a brief description of the data used in the study, followed by properties of the data as well as the descriptive statistics. Subsequent to the descriptive statistics are the presentation and interpretation of the results of all the estimated models, and the last section will cover diagnostic tests.

4.2 Data

In order to examine the risk-return relationship in the South African framework, this study uses the market index, namely FTSE/JSE Top 40, and the two sectoral indices namely the JSE Industrials and JSE Financials. The Top 40 index is a market capitalisation weighted index of the 42 largest companies listed on the JSE ranked by market capitalisation. Over the sample period, the Top 40 index represented an average of approximately 80 percent of the overall market value of the All Share Index as well as the most actively traded shares. The Top 40 index therefore serves as a good proxy for the South African share market. The JSE Financials index, which is also employed in the study, entails of all JSE-listed companies that belong to Industry Classification Benchmark (ICB) industry financials 8000. Again, the study uses JSE Industrial data. The Industrial index is a composite of all listed companies that do not belong to the ICB industries, financials, oil and gas (0001) and basic material (1000). The selection of data was solely based on data availability.

The data used in the study was obtained from Johannesburg stock exchange. This data focused on the period spanning from 1/1/2004 to 3/5/2017, giving us 4 872 observations for each index. This period is particularly interesting to study the risk return relationship because it covers the 2007 to 2009 financial crisis, which had an influence on volatility and a subsequent consequence on the returns of stocks. For the purpose of this study, it would have been more sensible to include data covering more than just one period of financial turmoil in order to adequately model the relationship and get an understanding of how different financial periods affect the relationship. However, I was limited in terms of obtaining daily data covering all the periods because of unavailability of daily data for periods before 2002.

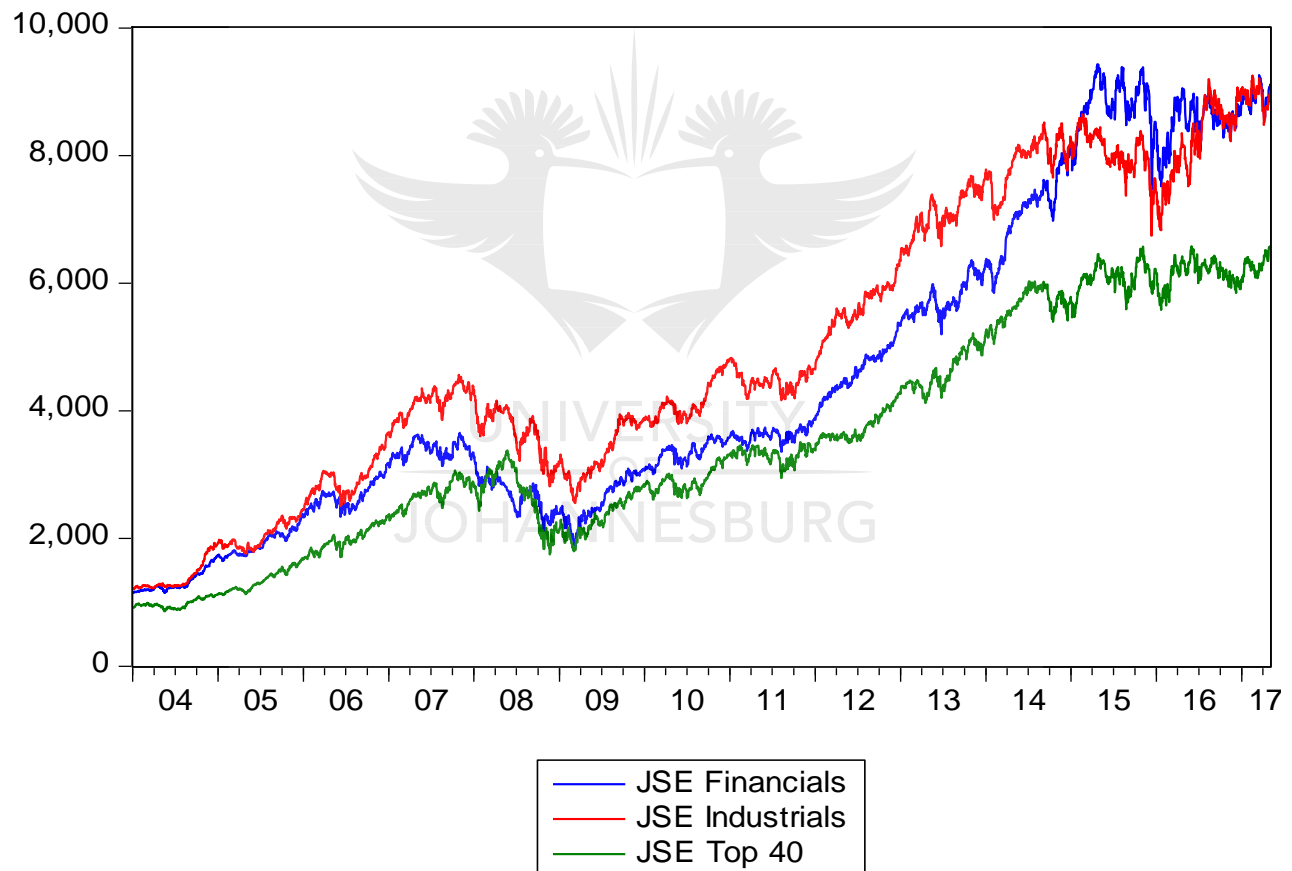
Although questions have been raised concerning the relevance of daily data because of non-trading days' effects, i.e. holidays and weekends, its use still proved to be more relevant relative to that of lower frequency data, given its ability to quickly assimilate new information. In support of the use of daily data are Mandimika and Chinzara (2010), who observed that increasing sampling frequency increases the accuracy of volatility estimates, and consequently is a good justification for using daily data for this study.

4.3 Properties of data

Figure 4.1 below is a graphical illustration of all the indices' closing prices over the period of study. From the graph, it is observed that around 2007, there was a sharp increase in prices, which was later followed by a dramatic fall in prices in 2008. The fall can be explained by the global financial crisis, which was responsible for declines in export markets, decreases in commodity prices and generated a slowdown in capital flows to developing countries. This as a result had an adverse impact on the South

African economy and thus spilled over to the performance of stocks in the JSE (Balchin, 2009). Stock prices started to improve in 2009, with a general upward trend until the end of 2013, with minor recoveries and drops. The trend observed in the prices of all indices resembles a random walk process, which is a typical feature of financial data. The price series do not revert back to their means; instead, they move away over the period and therefore we can make an assumption that all the prices series are non-stationary.

Figure 4.1: Time series plot of daily closing prices for JSE Financials, JSE Industrials and JSE Top 40



Then using the ADF test, data is formally tested for stationarity and the outcome indicates that indeed the series are non-stationary at levels as indicated in Table 4.1.

Stationarity is a necessary condition for the time series data, or else the variance will become unstable and shocks will be explosive (Brooks, 2002). As is common practice in standard empirical literature, in order to make the data stationary, the daily price series were transformed into continuous compounded returns using the following operation:

$$r_t = \log \left(\frac{P_t}{P_{t-1}} \right) = \log P_t - \log P_{t-1} \quad 4.1$$

where r_t is the log returns series, P_t and P_{t-1} are the current and previous period stock prices respectively for $t = 1, 2, \dots, \infty$. The new formulated return series have an advantage, in that they are free of unit roots and therefore are comparable. Since this transformation is expected to make the price series stationary, the new return series can now be modelled by a stochastic stationary process.

Figure 4.2: The time series plot of the logarithmic returns of JSE Top 40, Financials and Industrial Indices

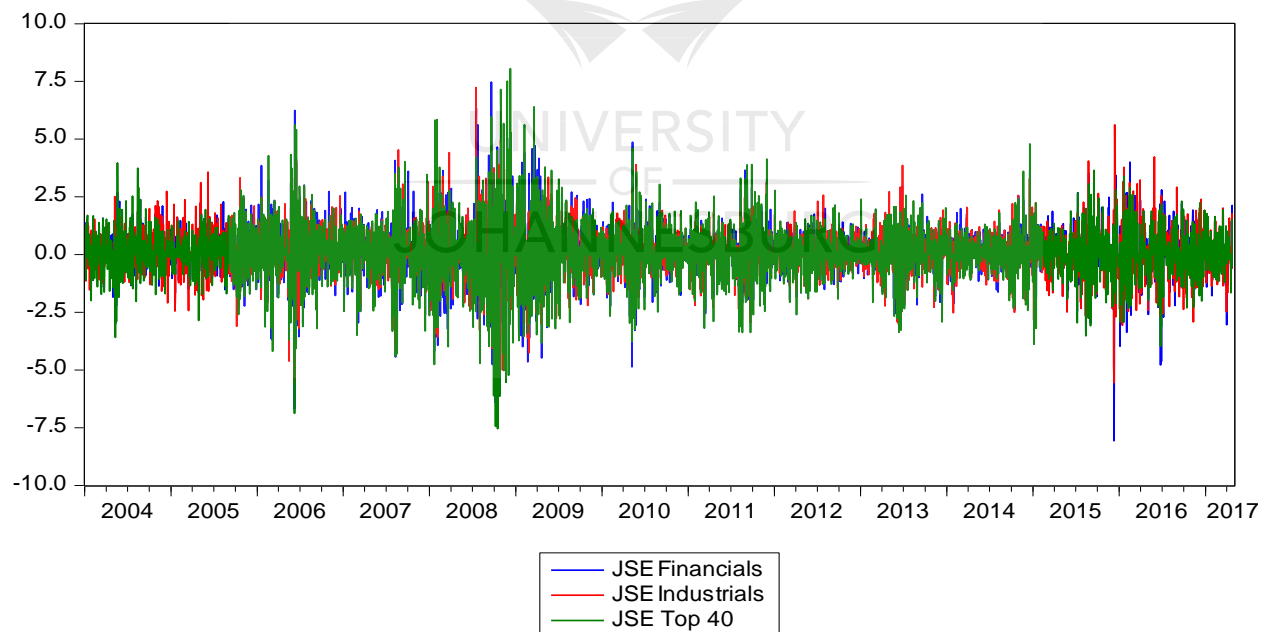


Figure 4.2 above is a plot of the logarithmic return series for the three indices, which all resemble a white noise process. All the return series for the indices have no upward or

downward trending behaviour; instead, they frequently revert back to their mean values. This implies that the returns series could now be stationary. However, these findings can only be substantiated by performing a formal equation for the test of stationarity, known as the augmented Dickey-Fuller (ADF) test, which is presented in equation 4.2 below. Under the ADF test, the null hypothesis tested is that the series has a unit root, where the null hypothesis is rejected if the ADF statistics are greater than the critical values. The results are presented in Table 4.1 below.

$$Y_t = \mu + \beta Y_{t-1} - \sum_{j=1}^p \alpha_j \Delta Y_{t-j} + \varepsilon_t \quad 4.2$$

Table 4.1: Augmented Dickey-Fuller test for unit root

INDEX	Augmented Dickey-Fuller test statistic	
	Levels	First difference
JSE Top 40	-0.445	-71.418
JSE Financials	0.051	-37.895
JSE Industrials	-0.664	-70.591

Critical values of ADF test at 1%, 5% and 10% are -3.431511, -2.861938 and -2.567024, respectively

The ADF test statistics for all the series at levels are observed to be statistically insignificant, with all the ADF test statistics less than critical values at all levels of significance, and therefore this leads us to the conclusion that all the price series are non-stationary. However, the ADF test statistic for the series in first difference terms is highly significant for all the indices, and therefore we make a conclusion that the returns for all the returns are stationary. Finally, from all the above plotted log returns series, volatility tends to be persistent; that is, periods of high movements in returns last for a while, followed by periods of low movement in returns also lasting for months and therefore producing volatility clusters (around 3rd quarter 2008, 2nd quarter 2010, 2nd and

3rd quarters 2011, 2nd quarter 2013 and 3rd quarter 2015). Volatility clustering is a typical feature observed in financial data. This feature was also observed by Mandelbrot (1963, P48), who described it as “large changes tend to be followed by large changes of either sign – or small changes followed by small changes in prices.” The presence of volatility clustering in the data therefore justifies the use of the GARCH family of models due to their ability to capture the time varying nature of volatility (Madimika & Chinzara, 2010)

4.3.1 Descriptive statistics

This section provides a starting point for our analysis as it outlines all the descriptive statistics tests performed on all the stock excess returns indices. The reported statistics are skewness, kurtosis, sample means, maximum, minimum, standard deviation and the Jarque-Bera (JB) test for normality.

The JB test statistic is calculated from both the skewness and kurtosis statistics. Its computation is as follows:

$$\frac{(T-k)}{6} \left(S_k^2 + \frac{(K_u-3)^2}{4} \right) \quad 4.3$$

where T denotes the total number of observations, k is representative of the number of estimated parameters, S_k is the skewness and K_u presents the kurtosis. The larger value of the test statistic implies the lower probability of the series following the normal distribution.

Under the JB test, we test for the null hypotheses that the returns are normally distributed. All the JB test statistics are statistically significant at the 1%, 5% and 10% level with all the p-values being less than 0. This, as a result, leads us to rejecting the null hypotheses which states that financial returns are follow a normally distribution, and

concluding that the indices' excess returns are not normally distributed. These findings correspond with the skewed returns and leptokurtic excess returns of the series reported earlier on.

Table 4.2: Summary of Descriptive Analysis of JSE Financials, Industrials and Top 40 indices returns

Index	Mean	Maximum	Minimum	Std. dev	Skewness	Kurtosis	Jarque-Bera	P-value
JSE Financials	0.048	7.473	-8.064	1.002	-0.042	9.826	9459.437	0.000
JSE Industrials	0.045	7.234	-5.555	0.931	-0.012	7.998	5069.296	0.000
JSE Top 40	0.046	8.054	-7.541	1.093	0.051	9.701	9116.653	0.000

Financial returns typically display a non-normal distribution and excess returns series under consideration are not an exception. Table 4.2 above displays summary descriptive statistics for excess daily return series for each index under consideration. Skewness measures the extent of asymmetry of a distribution about its mean value (Gujarati & Porter, 2009). A normal distribution has a skewness of zero and is symmetric about its mean. Kurtosis is a measure of how fat the tails of the distribution are (Gujarati & Porter, 2009). A normal distribution has a kurtosis of three. Both kurtosis and the skewness statistics for all indexes show a non-normal distribution in the daily returns for all indexes, with all skewness values being different from 0. Both the financial and industrial indices are negatively skewed, implying that most of the returns were less than the mean return, while the positively skewed Top 40 excess returns suggest that the majority of the series' excess returns are above the mean return. All kurtosis values for the return series are greater than 3. These values of kurtosis greater than three mean that the returns are all peaked around their mean value; they are leptokurtic, which is a phenomenon typical in financial time series data (Brooks, 2008).

The terminus a quo for our estimations was to decide on the suitable mean equation for each of the returns series. This was achieved by identifying the p and q orders of the ARIMA model to correct for autocorrelation that may be remaining in the differenced series by running the autocorrelation function (ACF) and the partial autocorrelation function (PACF). The Q-statistics obtained and the accompanying P-values associated with their ACF and PACF presented in Appendix 2 Correlograms 1 to 4 suggest that the null hypothesis of presence of random residuals should be rejected. Furthermore, from the ACF, the differenced series of all indices under consideration display a sharp cut-off at lag 1, with a negative spike, while the PACF decays sharply to zero. The negative spike suggests that one moving average (MA) term should be added to the model, since negative ACF and PACF suggest and MA (q) process. Furthermore, each of the series ACF abruptly cuts off at the first lag and PACF slowly decays. This suggests that each series can be modelled as an MA(1) process, which will be used to estimate time series models employing appropriate mean equations.

Before fitting the ARCH models to our series, we first have to examine if the variance of the error term of the mean model is heteroskedastic by using the ARCH-LM test. Under Engle's LM test, we are testing for autocorrelation in the squared residuals. Since the ARCH models have the form of an autoregressive model, Engle (1982) proposed the Lagrange multiplier (LM) test in order to investigate the presence of ARCH behaviour based on regression. To test for ARCH of order q , regress square of residuals $\hat{\varepsilon}_t^2 = \varphi_0 + \varphi_1 \varepsilon_{t-1}^2 + \dots + \varphi_q \varepsilon_{t-q}^2 + v_t$, v_t is iid. The test statistic is given by TR^2 , which is $X_{(q)}^2$ distribution on the null, where R is the sample multiple correlation coefficient computed from the regression of ε_t^2 on a constant and $\varepsilon_{t-1}^2, \dots, \varepsilon_{t-q}^2$ and T is the

sample size. The test is formulated as $H_0 : \varphi_0 = \varphi_1 = \dots = \varphi_q = 0$. We test for the joint null hypothesis that “all q lags of the squared residuals have coefficient values that are not significantly different from zero”, which is the case when no ARCH effects exist (Brooks, 2008) against H_1 : At least one φ is different from 0. $i = 1, 2, 3, \dots, q$ (Conditional variance is generated by ARCH process and is heteroskedastic). If the value observed from calculating the test statistic exceeds the critical from the χ^2 distribution, the null hypothesis is rejected.

The results are presented in Table 4.3 below. For all indices under consideration, we observe a high statistics value, which indicates that the null hypothesis of the presence of homoscedasticity is rejected and we conclude that there is a presence of ARCH effects. Due to the heteroskedastic nature of the variances in the error terms, the GARCH family of models, which is widely popular in modelling time-variant variance, should produce a good fit in the modelling of the variance of the error term. Because we want to model the leverage effect, which is a common characteristic in financial data, we will also use the variants of GARCH, namely E-GARCH-M and TARCH-M models.

Table 4.3: ARCH – LM test for heteroskedasticity

Index	Obs*R-squared	P-value
JSE Financials	130.9629	0.0000
JSE Industrials	63.56071	0.0000
JSE Top40	109.3643	0.0000

A conclusion on the risk-return relationship cannot be made on the basis of a comparison of returns means and standard deviation. As a result, formal tests to establish the risk-return relations will be conducted using GARCH-type models and they will be presented in the section that follows.

4.5 Results and interpretation

This section presents the main empirical results of the risk-return analysis. My attention was focused on the relationship between conditional volatility and expected market returns. It is important to note the results of this relationship because it has been an inconclusive point in the literature. This study cannot be directly compared with previous studies because of differences in the frequency and sample size. Additionally, the selection of data also includes the recent period of the global financial crisis (from October 2007), which is not included in any preceding study for South African markets.

Before estimations of all the models, diagnostic tests were performed on the residuals of the estimated models to establish whether the model has been correctly specified and guards against spurious inferences. The ARCH-LM test is used to test for heteroskedasticity in the standardised residuals of each of the estimated models. After running our test, we observe that for some models, such as TARCH (1,1)-M under all distributions and E-GARCH-(1,1)-M under the student-t and GED distributions for JSE Financials are a good fit as the ARCH effects have been removed. The TARCH (1,1)-M under GED and E-GARCH (1,1) under both student-t and GED distribution is a good fit for JSE Industrials. Lastly, JSE Top 40 is best modelled by TARCH (1, 1)-M under the normal distribution and E-GARCH (1, 1)-M under both the t-distribution and GED.

The GARCH (p, q)-M, E-GARCH (p, q)-M and TARCH (p, q)-M models are estimated for the three indices under consideration. Before the processing of all the models, the first step, based on the Box Jenkins methodology, was to identify the suitable model for each return series. As mentioned earlier, the Q-statistics obtained and the accompanying P-values associated with their ACF and PACF function presented in

Appendix 2 Correlograms 1 to 4 suggest the null hypothesis of presence or random residuals. Furthermore, from the ACF, the differenced series of all indices under consideration display a sharp cut-off at lag 1, with a negative spike, while the PACF decays sharply to zero. The negative spike suggests that one moving average (MA) term should be added to the model, since negative ACF and PACF suggest an MA (q) process. Furthermore, each of the series ACF abruptly cuts off at the first lag and PACF slowly decays. This suggests that each series can be modelled as an MA(1) process, which will be used in estimating time series models employing appropriate mean equations. In addition, all the coefficients of the estimated MA(1) for the series have significant t-statistics, which are greater than two in absolute terms (see table in appendices). The second step was to carry out an ARCH-LM test to ensure that the process is aligned with the suggested GARCH (p, q)-M and its variants. The test results, as presented in the Table 4.3, indicate the presence of ARCH effects in all the three indices to lag 1. For simplicity, the returns of the series are modelled as GARCH (p, q)-M, E-GARCH (p, q)-M and TARCH (p, q)-M. According to Asteriou and Hall (2007), GARCH (1,1)-M, as opposed to higher order GARCH-M (p,q) models, usually perform well and are simpler to estimate given the few numbers of parameters; furthermore, Engle (1982) states that GARCH (1,1) is the most important and simultaneously the simplest of the category of volatility models.

Our estimates show that all indices have a highly significant persistence measure β_i for E-GARCH (1,1)-M, which, in most cases, ranges between 0.9 and 1 for both the E-GARCH (1,1)-M for TARCH (1,1)-M models. Persistence is highly significant as the measure $2(1 - \alpha_1 - \beta_1)$ also ranges from 0.9 to 0.1 in most cases. However, there are

a few exceptions, where the β_j parameters are small, such as E-GARCH (1,1)-M, under both the student-t distribution, and the GED distributions are -0.1109 and -0.2023. Moreover, in the case of the GARCH (1,1)-M models, the persistence parameter ($\alpha_i + \beta_i$) in the estimated GARCH models is very close to 1 in most instances being between 0.9 and 1. However, just as with the EGARCH (1,1)-M and TARCH (1,1)-M model, we have persistence values that are not close to 1, but are either very large or too small. Under the normal distribution, student-t and GED distributions' persistence values are 50.72, 44.87 and 129.68 for JSE Top 40, JSE Industrials and JSE Financials, respectively. These high persistence values greater than 1 are an indication that shocks have everlasting effects on conditional variance or return, thereby reflecting persistence in variance.

As mentioned earlier in Chapter 3, the leverage effect coefficient prevails in the E-GARCH (1,1)-M and TARCH (1,1)-M models. Looking at the estimations of the two models, we observe consistency with theory. The models significantly detain the leverage effect in all the indices, with the exception of all indices under the GED distribution, which have 0 leverage coefficient in most cases for both E-GARCH (1,1)-M and TARCH (1,1)-M models. The E-GARCH (1,1)-M model leverage coefficient is negative and significant, as well as the TARCH (1,1)-M displaying a positive and significant leverage effect. In the models where the sign was expected, i.e. positive coefficient for TARCH (1,1)-M and negative for E-GARCH(1,1)-M, the implication is that there is an asymmetric effect, and therefore an unanticipated decline in price results in volatility increasing by more than an unanticipated increase in the price of similar degree (Brooks, 2008). The observed presence of leverage effect in the return series

implies that of volatility has asymmetric effects, and therefore the use of GARCH-M models alone would not have been sufficient in modelling the returns; hence, the use of the EGARCH(1,1)-M and TARCH(1,1)-M models is justified.

From the estimated results presented in Tables 1 to 6 in Appendix 1, it has been observed that MA(1) coefficients are significant for all indices under the two distributional assumptions, i.e. normal and GED distribution. This implies that the hypothesis of successive log price changes having linear independence is strongly rejected for the indices, and therefore it can be concluded that market inefficiency will be affected by volatility. The most significant parameter, particularly for the purpose of our study, is the risk aversion parameter denoted by φ in equation 20, Chapter 3, as it allows us to assess the relationship between risk and return.

From Tables 1 to 6 in Appendix 1, it is evident that the risk-return relationship does give us mixed results with some models giving us positive and significant results, while others showed a negative and insignificant relationship.

Generally, the GARCH (1,1)-M model shows confirms the existence of a positive risk premium for the three indices. However, there are exceptions, where the risk coefficients for Top 40, Industrials and Financials are positive but insignificant. We observe the following coefficient values: 0.0010 (0.9760), 0.0002 (0.9934) and 0 (0.9913), with p-values in parenthesis. These positive findings are in line with those observed by Chiang Zhang (2018) who found a significantly positive relationship in the Chinese equity market based on the TARCH-M model. They found a positive relationship in both the aggregate and the sectoral markets while controlling for sentiment and liquidity.

The E-GARCH (1,1)-M estimates, on the other hand, show different prediction patterns with respect to the risk-return relationship. We observe that none of the models show a positive and significant coefficient; instead, for JSE Top 40 under normal, student-t and GED distributions, the coefficients are 0.0480 (0.2042), -0.0001(0.9661), -2.2394 (0.0000), respectively, with p-values in parenthesis. JSE Industrial similarly showed positive and insignificant coefficients. The normal distribution, student-t and GED distribution produced -0.0379 (0.4589), 0.0003 (0.9719) and -0.1417(0.0000).

Similar to the E-GARCH (1,1)-M, the TARCH (1,1)-M model produced insignificantly positive, and significantly negative risk-return coefficients. The JSE Top 40, under the normal, student-t and GED, have the coefficients 0.1056(0.0170), 0.0185 (0.1475) and -0.0003(0.0326). For JSE Industrials, the coefficients for the normal, student-t and GED distributions are 0.0002(0.9796), -0.0160(0.8538) and 0.0028(0.0000) and for JSE Financials the estimated coefficients are 0.0374 (0.4152), 0.0020(0.7409) and TARCH (1,1)-M under GED distribution is the only model that has a positively significant coefficient.

The models that accommodated for the impact of financial crises on volatility by including the dummy variables were also estimated. Generally, it has been observed from this model that the dummy variables have a significant positive impact on volatility. For example, for the Top 40 GARCH (1,1)-M model under GED, the coefficient is 0.0096 (0.0000), and for E-GARCH under student-t distribution the coefficient is 1.1582 (0.0000). Similar patterns are observed with all the indices under the different distribution assumptions. This implies that periods of financial instability influenced volatility in the stock market. However, the inclusion of the financial period had little or

no impact on our risk-return parameter, as the change was very insignificant and did not change the sign of the coefficient.

From the above results, the main conclusion made is that the E-GARCH (1,1)-M and TARCH(1,1)-M framework fail to sufficiently display favourable evidence of the risk-return trade-off in the South African stock market. The two models leave us with inconclusive evidence about either the sign or the significance of the relationship. For all the indices, the models under the GED distribution have a significant but negative trade-off, with the exclusion of TARCH (1,1)-M under GED for all indices. These results obtained are inconsistent with the theoretical model from the theoretical underpinning on which estimations are founded. Following Merton's ICAPM, a positive and significant risk-return relationship was expected. Our results are in line with those of Madimika (2009), who observed that volatility is not a priced factor in the South African stock market.

4.6 Conclusion

The presence of a positive risk premium observed in GARCH (1,1)-M is in consensus with empirical literature (French *et al.*, 1987; Campbell & Hentschel, 1992). Since risk-neutral investors have a linear risk-return relationship, the implication that we get from the GARCH(1,1)-M estimations is that such investors will tend to invest indices, since they have a positive risk-return relationship. In contrast, the negative and robust relationship that is observed when employing the E-GARCH(1,1)-M and TARCH(1,1)-M is supported by the findings of Fraser and Power (1997), who did not find evidence of a positive risk premium in nine emerging markets. The lack of a positive and significant

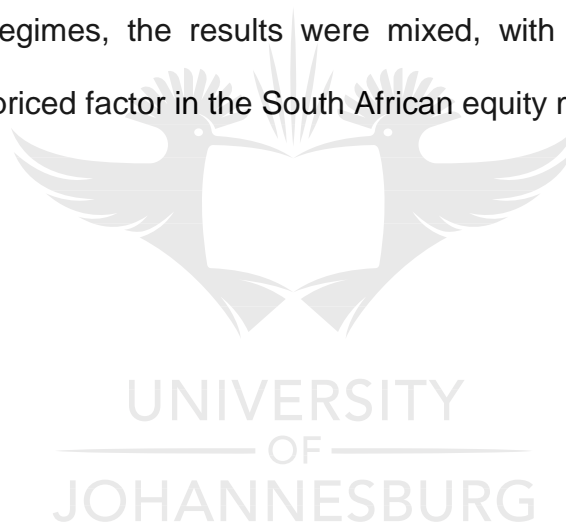
risk-return relationship on majority of the stocks on the South African equity markets violates the prediction of many asset pricing models, such as the CAPM and the APT. According to Madimika (2009), the negative relationship observed in the returns could be attributed to the currency used in calculating the return. Since investing domestically does not expose investors to foreign exchange risk, when investments are valued in the local currency, the premium would be negative, whereas, if the returns were converted to a foreign currency such as dollars, this would imply that risk would not only incorporate volatility in the local markets, but also incorporate foreign exchange risk. This theory is also supported by Koutmos *et al.* (1993), who found a statistically significant relationship between risk and returns when converting returns from local currency to US dollars.

It is possible that the negative relationship observed in most of the returns on South Africa's equity markets could be attributable to the currency used to measure the returns. Since domestic investors are not faced with foreign exchange risk (when returns are measured in Rands), the premium would be negative and significant, while, if the returns were converted to a foreign currency, this would imply that risk would not only incorporate volatility in the local markets, but also incorporate foreign exchange risk. This finding is supported by Koutmos *et al.* (1993), who detected a statistically significant relationship between risk and return when returns were converted from the domestic currency to US dollars.

Another reason that could be attributed to the negative risk-return relationship is the inclusion of the intercept in the conditional mean equations. Lanne and Saikkonen (2006) have observed that, although based on the ICAPM, there is no theoretical

justification to include the intercept in the conditional mean model. Most empirical studies examining the risk-return trade-off include the intercept in the model. They also failed to find a risk-return trade-off that was positive in the US stock returns when the intercept is included in the model.

Summing up, this chapter demonstrated the analysis of the risk return relationship in the South African equity market. Firstly, descriptive statistics of the data were analysed on the dataset and the data exhibited properties aligned with financial data. Using GARCH-M (1,1), E-GARCH-M (1,1) and TARCH-M (1,1) augmented with dummy variable to account for different regimes, the results were mixed, with most estimated models showing risk as a non-priced factor in the South African equity market.



CHAPTER 5: CONCLUSION AND POLICY RECOMMENDATIONS

5.1 Conclusion

The global financial crisis of recent years has drawn special attention, to academics and regulators similarly, the significance of understanding the causes and effects of volatility, especially in the capital and financial markets, as well as in the broader economy. Our study's objective was to empirically test the risk-return relationship within the South African capital markets using the GARCH-M models; further to this, the study was extended to investigate the risk-return relationship.

Daily data for FTSE/JSE Top 40, JSE Industrial and JSE Financial covering the period January 2004 to May 2017 was used. The family of GARCH-models was estimated under three error distributional assumptions. This research was aimed at bridging the gap in the existing literature by evaluating the risk-return relationship accounting for different regimes in the market by employing the family of GARCH-M models with the conditional variance augmented with dummy variable representing the period of financial turmoil.

The initial step for the analysis was to explore the relevant theoretical and empirical literature. The theoretical literature was explored with the purpose of understanding the relevant models on the risk return relationship, as well as the results that were previously observed on the topic. Following the discussion of the theoretical literature was the empirical literature on developed markets and the South African stock market. The empirical literature showed that there is no consensus on the existence of a premium for taking more risk.

In order to address the objectives of this study, E-GARCH-M and T-GARCH-M models were estimated, as well as three distributional assumptions, namely normal, student-t, and the GED. The results indicated that, for all indices, volatility is persistent and asymmetric. The findings of this study were generally in line with observations made by other authors on the risk-return relationship, which indicated that volatility is not a priced factor, even after accounting for periods of financial turmoil, as indicated in Chapter 4.

5.2 Limitations

A major limitation in the study was that we had to use data only covering one period of the financial crisis and could not include more, e.g. the Asian financial crises, as daily data covering that period was not available to the author. Research has shown that the inclusion of different of different financial crises periods could potentially change the conclusion of the study, and thereby improve results.

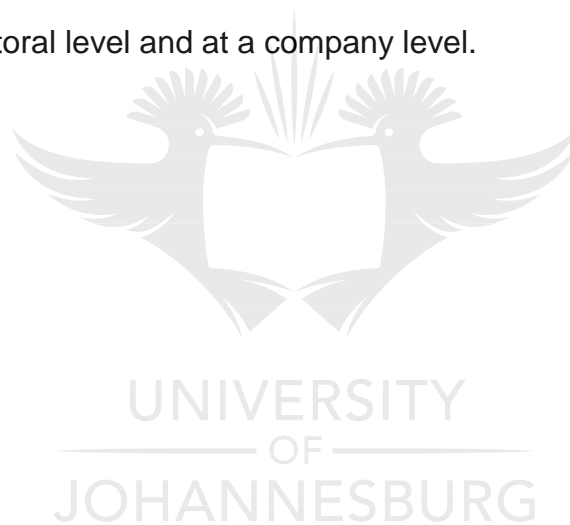
5.3 Policy and implications

The outcomes of this study have important inferences for investment and policy. The observation made in Chapter 4, through E-GARCH (1,1)-M and TARARCH (1,1)-M, that investors are not compensated for taking more risk, would have an implication on dynamics to take into account when making an investment decision. When investing, investors and policymakers need to take into consideration the general increase in volatility and how differently it affects industries and sectors. For investors, it would be meaningful to capitalise on sectors and industries that are commonly less volatile, especially if this general increase in volatility is not associated with increasing returns. For policymakers, increasing volatility is an issue as it may result in capital outflow in

large quantities, which could cause financial instability and subsequently trigger instability on a macroeconomic level.

5.4 Areas for further research

While this study estimated the GARCH-M model and its variants, including the intercept in the equation for conditional variance, we should consider excluding it in the future, which is an option that was not in this current study. Furthermore, since this study was mainly done for JSE Top 40, which is representative of the whole market and two industrial indices and sectoral level, it could be worthwhile to extend this study to sectoral and super sectoral level and at a company level.



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Appendix 1

Table 6.1: The risk-return relationship under GARCH(1,1)-M specification

Distribution	JSE top 40			
	Risk premium	MA(1)	Persistence	SIC
Normal distribution	0.1102	-0.0052	0.9908	2.7301
<i>P-Value</i>	0.0246	0.7275		
Student-t distribution	0.001	-0.1187	50.7278	2.5765
<i>P-Value</i>	0.976	0.2687		
GED	0	0	0.9748	-1.8789
<i>P-Value</i>	0	0		
Distribution	JSE industrials			
	GARCH coefficient	MA(1)	Persistence	SIC
Normal distribution	0.1556	0.015511	0.9854	2.5211
<i>P-Value</i>	0.7811	0.2922		
Student-t distribution	0.0002	0.0028	14.8203	2.3492
<i>P-Value</i>	0.9934	0.7922		
GED	0.0556	0	0.5882	0.2377
<i>P-Value</i>	0	0.0022		
Distribution	JSE financials			
	GARCH coefficient	MA(1)	Persistence	SIC
Normal distribution	0.0539	0.0068	0.9893	2.5765
<i>P-Value</i>	0.259	0.6332		
Student-t distribution	0	-0.0113	129.6805	2.3902
<i>P-Value</i>	0.9913	0.2857		
GED	-0.02552	0	0.6459	0.6833
<i>P-Value</i>	0	0		

Table 6. 2: The risk-return relationship under E-GARCH(1,1)-M specification

Distribution	JSE top 40				
	Risk premium	MA(1)	Persistence	Leverage coefficient	SIC
Normal distribution	0.048	-0.0004	0.9891	-0.0854	2.7015
<i>P-Value</i>	<i>0.2042</i>	<i>0.9805</i>		<i>0</i>	
Student-t distribution	-0.0001	-0.006171	-0.0379	-2.4448	2.5982
<i>P-Value</i>	<i>0.9661</i>	<i>0.6152</i>		<i>0.9656</i>	
GED	-2.2394	0	-0.0393	0	1.0123
<i>P-Value</i>	<i>0</i>	<i>0</i>		<i>0.3744</i>	
Distribution	JSE industrials				
	Risk premium	MA(1)	Persistence	Leverage coefficient	SIC
Normal distribution	-0.0379	0.2244	0.984	-0.0631	2.5113
<i>P-Value</i>	<i>0.4589</i>	<i>0.1181</i>		<i>0</i>	
Student-t distribution	0.0003	0.0093	-0.1171	-1.8771	2.352
<i>P-Value</i>	<i>0.9718</i>	<i>0.4532</i>		<i>0.971</i>	
GED	-0.1417	0	-0.2023	0	0.3325*
<i>P-Value</i>	<i>0</i>	<i>0</i>		<i>0.0001</i>	
Distribution	JSE Financials				
	Risk premium	MA(1)	Persistence	Leverage coefficient	SIC
Normal distribution	0.0324	0.006	0.9873	-0.0736	2.5649
<i>P-Value</i>	<i>0.4505</i>	<i>0.6646</i>		<i>0</i>	
Student-t distribution	-0.0006	-0.0119	0.9882	-0.2045	2.3894
<i>P-Value</i>	<i>0.962</i>	<i>0.2597</i>		<i>0</i>	
GED	-1.3299	0	0.9111	0	1.0581
<i>P-Value</i>	<i>0</i>	<i>0.0002</i>		<i>0.8414</i>	

Table 6.3: The risk-return relationship under TARCH(1,1)-M specification

Distribution	JSE top 40				
	Risk premium	MA(1)	Persistence	Leverage coefficient	SIC
Normal distribution	0.1056	- 0.001294	0.9465	0.0885	2.7016
<i>P-Value</i>	<i>0.017</i>	<i>0.931</i>		<i>0</i>	
Student-t distribution	0.0185	- 0.010095	0.9487	1.0979	2.5654
<i>P-Value</i>	<i>0.1475</i>	<i>0.3498</i>		<i>0.173</i>	
GED	-0.0003	0	0.9078	-0.0004	-0.872
<i>P-Value</i>	<i>0.0326</i>	<i>0.2423</i>		<i>0</i>	
Distribution	JSE industrials				
	Risk premium	MA(1)	Persistence	Leverage coefficient	SIC
Normal distribution	0.0002	0.015	0.9315	119.4741	2.3436
<i>P-Value</i>	<i>0.9796</i>	<i>0.3085</i>		<i>0.9896</i>	
Student-t distribution	-0.01	0.0042	0.9381	0.0714	2.5046*
<i>P-Value</i>	<i>0.8538</i>	<i>0.6912</i>		<i>0</i>	
GED	0.0028	0	0.8582	-0.0001	- 0.4633*
<i>P-Value</i>	<i>0</i>	<i>0</i>		<i>0</i>	
Distribution	JSE financials				
	Risk premium	MA(1)	Persistence	Leverage coefficient	SIC
Normal distribution	0.0374	0.0044	0.9349	0.0744	2.5611
<i>P-Value</i>	<i>0.4152</i>	<i>0.7551</i>		<i>0</i>	
Student-t distribution	0.002	-0.0095	0.9405	3.6097	2.3838
<i>P-Value</i>	<i>0.7409</i>	<i>0.3687</i>		<i>0.6252</i>	
GED	0.0423	0	0.8159	0.0001	0.0116
<i>P-Value</i>	<i>0</i>	<i>0</i>		<i>0</i>	

DIAGNOSTIC TESTS

JSE TOP 40 INDEX

Table 6.4: TARCH-M heteroskedasticity test ARCH normal distribution

F-statistic	0.68077	Prob. F(1,4868)	0.4094
Obs*R-squared	0.68095	Prob. chi-square(1)	0.4093

Table 6.5: TARCH-M heteroskedasticity test ARCH student-t distribution

F-statistic	0.00113	Prob. F(1,4868)	0.9732
Obs*R-squared	0.00113	Prob. chi-square(1)	0.9731

Table 6.6: TARCH-M heteroskedasticity test ARCH GED

F-statistic	168.582	Prob. F(1,4868)	0
Obs*R-squared	163.006	Prob. chi-square(1)	0

E-GARCH-M

Table 6.7: E-GARCH-M heteroskedasticity Test ARCH normal distribution

F-statistic	0.11454	Prob. F(1,4868)	0.735
Obs*R-squared	0.11458	Prob. chi-square(1)	0.735

Table 6.8: E-GARCH-M heteroskedasticity test ARCH student-t distribution

F-statistic	13.4741	Prob. F(1,4868)	0.0002
Obs*R-squared	13.4424	Prob. chi-square(1)	0.0002

Table 6.9: E-GARCH-M heteroskedasticity test ARCH GED distribution

F-statistic	168.767	Prob. F(1,4868)	0
Obs*R-squared	163.179	Prob. Chi-Square(1)	0

JSE FINANCIALS

TARCH-M

Table 6.10: Heteroskedasticity ARCH normal distribution

F-statistic	4.6847	Prob. F(1,4868)	0.0305
Obs*R-squared	4.68212	Prob. chi-square(1)	0.0305

Table 6.11: Heteroskedasticity test ARCH student-t distribution

F-statistic	13.7506	Prob. F(1,4868)	0.0002
Obs*R-squared	13.7175	Prob. chi-square(1)	0.0002

Table 6.12: Heteroskedasticity test ARCH GED

F-statistic	220.267	Prob. F(1,4868)	0
Obs*R-squared	210.819	Prob. chi-square(1)	0

JSE FINANCIALS**E-GARCH-M****Table 6.13: Heteroskedasticity test ARCH normal distribution**

F-statistic	1.76198	Prob. F(1,4868)	0.1844
Obs*R-squared	1.76207	Prob. chi-square(1)	0.1844

Table 6.14: Heteroskedasticity test ARCH Student-t distribution

F-statistic	14.3411	Prob. F(1,4868)	0.0002
Obs*R-squared	14.3048	Prob. chi-square(1)	0.0002

Table 6.15: Heteroskedasticity test ARCH GED

F-statistic	119.039	Prob. F(1,4868)	0
Obs*R-squared	116.245	Prob. chi-square(1)	0

JSE INDUSTRIALS**TARCH-M****Table 6.16: Heteroskedasticity test ARCH normal distribution**

F-statistic	0.52244	Prob. F(1,4868)	0.4698
Obs*R-squared	0.52259	Prob. chi-square(1)	0.4697

Table 6.17: Heteroskedasticity test ARCH student distribution

F-statistic	2.50427	Prob. F(1,4868)	0.1136
Obs*R-squared	2.50401	Prob. chi-square(1)	0.1136

Table 6.18: Heteroskedasticity test ARCH GED

F-statistic	118.879	Prob. F(1,4868)	0
Obs*R-squared	116.092	Prob. chi-square(1)	0

E-GARCH-M**Table 6.19: Heteroskedasticity test ARCH normal distribution**

F-statistic	1.76198	Prob. F(1,4868)	0.1844
Obs*R-squared	1.76207	Prob. chi-square(1)	0.1844

Table 6.20: Heteroskedasticity test ARCH student-t

F-statistic	14.3411	Prob. F(1,4868)	0.0002
Obs*R-squared	14.3048	Prob. chi-square(1)	0.0002

Table 6.21: Heteroskedasticity test: ARCH GED

F-statistic	119.039	Prob. F(1,4868)	0
Obs*R-squared	116.245	Prob. chi-square(1)	0

Table 6.22: Regression results of MA (1) models for all series

Series		MA(1) coefficient	Std. error		t-Statistics	P-value
JSE top 40	-	0.4480	0.1469	-	3.0499	0.0037
JSE financials	-	0.3430	0.1710	-	2.0051	0.0049
JSE industrials	-	0.4714	0.1614	-	2.9208	0.0047

Appendix 2

CORRELOGRAMS

Correlogram 1: Logarithm of daily returns JSE top 40

Sample: 1/01/2004 5/03/2017										
Included observations: 4870										
Autocorrelation			Partial correlation				AC	PAC	Q-stat	P-value
****			****			1	-0.493	-0.493	1183.2	0.000
			**			2	-0.005	-0.327	1183.3	0.000
			**			3	0.005	-0.236	1183.5	0.000
			*			4	-0.008	-0.193	1183.8	0.000
			*			5	-0.014	-0.185	1184.7	0.000
			*			6	0.031	-0.128	1189.3	0.000
			*			7	-0.016	-0.113	1190.5	0.000
			*			8	-0.031	-0.154	1195.1	0.000
			*			9	0.047	-0.104	1205.9	0.000
			*			10	-0.023	-0.105	1208.5	0.000
			*			11	0.007	-0.092	1208.8	0.000
			*			12	-0.006	-0.095	1208.9	0.000
						13	0.021	-0.056	1211.1	0.000
			*			14	-0.029	-0.074	1215.1	0.000
			*			15	0.011	-0.073	1215.7	0.000
			*			16	0.001	-0.067	1215.7	0.000
						17	0.013	-0.034	1216.6	0.000
						18	-0.025	-0.056	1219.5	0.000
						19	0.006	-0.062	1219.7	0.000
			*			20	-0.001	-0.068	1219.7	0.000
						21	0.004	-0.064	1219.8	0.000
						22	0.018	-0.033	1221.4	0.000
						23	-0.015	-0.029	1222.5	0.000
						24	-0.021	-0.065	1224.6	0.000
						25	0.029	-0.042	1228.7	0.000
						26	-0.004	-0.033	1228.8	0.000
						27	-0.009	-0.038	1229.2	0.000
						28	-0.007	-0.059	1229.4	0.000
						29	0.028	-0.020	1233.3	0.000
						30	-0.021	-0.024	1235.4	0.000
						31	0.009	-0.014	1235.8	0.000
						32	-0.004	-0.013	1235.9	0.000
						33	0.013	0.022	1236.7	0.000
						34	-0.047	-0.042	1247.5	0.000
						35	0.030	-0.044	1251.8	0.000
						36	0.002	-0.039	1251.8	0.000

Correlogram 2: Logarithm of daily returns JSE industrials

Sample: 1/01/2004 5/03/2017										
Included observations: 4870										
Autocorrelation			Partial correlation				AC	PAC	Q-stat	P-value
****			****			1	-0.478	-0.478	1111.2	0.000
			**			2	-0.033	-0.338	1116.6	0.000
			**			3	0.033	-0.219	1121.9	0.000
			**			4	-0.038	-0.208	1129.0	0.000
			*			5	0.017	-0.168	1130.4	0.000
			*			6	-0.020	-0.178	1132.3	0.000
			*			7	0.040	-0.112	1140.2	0.000
			*			8	-0.032	-0.124	1145.1	0.000
			*			9	0.011	-0.108	1145.7	0.000
			*			10	0.008	-0.088	1146.0	0.000
			*			11	-0.007	-0.077	1146.2	0.000
			*			12	-0.010	-0.090	1146.8	0.000
			*			13	0.009	-0.082	1147.2	0.000
						14	0.014	-0.053	1148.2	0.000
			*			15	-0.023	-0.067	1150.7	0.000
			*			16	0.008	-0.066	1151.0	0.000
						17	0.002	-0.063	1151.0	0.000
						18	0.005	-0.049	1151.1	0.000
						19	0.011	-0.019	1151.7	0.000
						20	-0.022	-0.033	1154.0	0.000
			*			21	-0.024	-0.088	1156.7	0.000
						22	0.058	-0.024	1173.3	0.000
						23	-0.023	-0.017	1175.9	0.000
			*			24	-0.032	-0.066	1180.8	0.000
						25	0.042	-0.034	1189.5	0.000
						26	-0.025	-0.051	1192.5	0.000
						27	0.011	-0.042	1193.0	0.000
						28	-0.004	-0.047	1193.1	0.000
						29	0.023	-0.008	1195.8	0.000
						30	-0.034	-0.036	1201.6	0.000
						31	0.030	0.002	1205.9	0.000
						32	-0.031	-0.038	1210.6	0.000
						33	0.030	0.001	1215.1	0.000
			*			34	-0.049	-0.067	1226.8	0.000
						35	0.041	-0.043	1234.9	0.000
						36	0.014	-0.015	1235.8	0.000

Correlogram 3: Logarithm of daily returns JSE financials

Sample: 1/01/2004 5/03/2017							
Included observations: 4870							
Autocorrelation	Partial correlation		AC	PAC	Q-stat	P-value	
***	***	1	-0.475	-0.475	1099.7	0.000	
	**	2	-0.036	-0.338	1106.0	0.000	
	**	3	0.034	-0.218	1111.5	0.000	
	**	4	-0.045	-0.217	1121.2	0.000	
	*	5	0.020	-0.177	1123.2	0.000	
	*	6	0.007	-0.144	1123.4	0.000	
	*	7	-0.003	-0.121	1123.5	0.000	
	*	8	-0.019	-0.139	1125.2	0.000	
	*	9	0.013	-0.129	1126.0	0.000	
	*	10	0.013	-0.099	1126.8	0.000	
	*	11	-0.012	-0.096	1127.5	0.000	
	*	12	0.010	-0.078	1128.0	0.000	
		13	0.014	-0.036	1128.9	0.000	
		14	-0.037	-0.064	1135.5	0.000	
		15	0.016	-0.058	1136.7	0.000	
	*	16	-0.018	-0.089	1138.4	0.000	
		17	0.033	-0.049	1143.6	0.000	
		18	-0.021	-0.062	1145.8	0.000	
		19	0.021	-0.028	1147.9	0.000	
		20	-0.027	-0.053	1151.5	0.000	
		21	0.001	-0.065	1151.5	0.000	
		22	0.026	-0.037	1154.7	0.000	
		23	-0.005	-0.016	1154.8	0.000	
	*	24	-0.046	-0.085	1165.2	0.000	
		25	0.064	-0.022	1185.3	0.000	
		26	-0.043	-0.052	1194.4	0.000	
		27	0.015	-0.041	1195.5	0.000	
		28	-0.002	-0.052	1195.5	0.000	
		29	0.004	-0.039	1195.6	0.000	
		30	0.010	-0.017	1196.1	0.000	
		31	-0.014	-0.019	1197.1	0.000	
		32	0.001	-0.023	1197.1	0.000	
		33	0.020	0.021	1199.0	0.000	
	*	34	-0.065	-0.072	1219.6	0.000	
		35	0.051	-0.052	1232.5	0.000	
		36	0.013	-0.019	1233.3	0.000	